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Nambar ngaihnawm pahnih – π (pi) leh φ (phi)

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INTRODUCTION: NAMBAR SET CHI HRANG HRANGTE - N, W, Z, Q, R & C

1. Natural numbers, N

A mâwl thei ber chu 'thil zât chhiarna' ti mai ila - a thua, *pakhat, pahnih, pathum*… kan tih mai, a nambara ziaka 1, 2, 3, … hi a bul ṭan nan hmang phawt ta ila. Hetianga nambar chhinchhiahna (symbols) mumal tak hman tûr kan lo neih dân hi chhui tham leh sawi tham a awm a. Chu lam chu kan luhchilh lo vang a. A chunga kan han sawi khi chu mi zawng zawng hriat saah kan ngai mai ang.

Kan chhiar duh chu mihring an ni emaw, ran an ni emaw, thei emaw pawisa emaw arsi emaw pawh ni se, a nambara kan ziak chuan 1, 2, 3, … tih hi kan hmang vek mai. Hei hi Mizo ṭawng chuan 'tam lam chhiarna nambar' kan ti mai ang a, Sapṭawng chuan '*counting numbers*' emaw '*cardinal numbers*' emaw an ti a. Mathematics lam mi pawhin heng hmingte hi an pawm tho nâ-in 'natural numbers' tih hming hi an thlang deuh bik tlângpui a. Hei hi nambar tlânglâwn leh hriat lâr ber a ni.

Ngaihtuahna hman hmasak ber nan: natural nambar-te hi a vaiin eng zât nge? Han chhiar char char ta ila, Mizo ṭawng chuan *sâwm, za, sang, sing* kan ti chho zel ang a; *nuai, maktaduai, vaibêlchhia* kan ti chho leh ang a, *tlûklehdingâwn* bâk chu nambar hming kan nei sêng tawh lo a ni mai thei. Chhiar tâwp tih ni a awm thei lo tih a chiang sa mai; chuvangin 'chhiar sên loh' a ni tih a chiang reng mai, Sapṭawng kan hawh ang a, 'infinity' a ni kan ti mai ang (the set of natural numbers is infinite). Chhiar tâwp sên loh 'infinity' chu a chhinchhiah nan ∞ hi kan hmang ang a. Natural nambar 'set' chu N hian kan chhinchhiah ang, hetiangin:

$$N = \{1, 2, 3, \ldots. n, \ldots\ldots, \infty\}$$

2. Whole numbers, W

Chuti a nih chuan, zawhna awm leh ta chu: "Natural nambarte hi nambar awm thei zawng zawng a ni tihna em ni? Hetah hian nambar zawng zawng a kim vek em ni?" A chhânna chu kan hre sa vek tawh tho bawk a: "Kim awzâwng lo mai!" tih hi a ni. A hmasa berah chuan 0 (zero), mathematician-te tâna a pawimawh ber tih theih hial chu a la tel lo tlat. Zero chu eng nge ni a? Chhânna awlsam ber chu 'engmahlo' = "nambar amah leh amah inpaih ral" kan ti mai ang chu. Entirnan: 3-3 = 0. Tichuan nambar kan han sawi tawh hote chu heti hian kan ziak ang 0, 1, 2,

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3, … Hei hi natural nambar leh 0 a ni a; 'the set of whole numbers' kan vuah ang a, a 'set' chhinchhiah nan W kan hmang a.

W = {0, 1, 2, 3, ….., n, ….. ∞}

3. Integers, Z

Ti hian a kim thei tawh em? A la kim chuang lo. Chhiarkawp mâwl ber: belh (+) leh paih (-) kan chawh chhuah dâwn chuan, nambar chi dang a awm tih a lo lang. Entirnan: 7 - 5 = 2 a ni a; 10 - 6 = 4 a ni a. A ṭha e. A nih leh 10 - 15 = ? Sâwm aṭanga sâwmpanga paih a awm thei lo, kan ti mai dâwn em ni?

Dâwrah naupang Rs 10 man thil lei turin kan tir a, mahse Rs 15 man a lo ni tawh si a, a rawn hâwn ta tho va; eng nge a awmzia ni ta? Rs 5 a rawn ba haw a nih chu! Chu chu engtin nge a nambarin kan ziak tak ang? 'minus' or 'negative' nambar kan la mamawh a nih chu; hetiangin: 10 - 15 = - 5. Tichuan, 'the set of whole numbers' kan tih pawh khi a lo la kim tâwk lo va, 'negative nambar' kan neih tel a ngai a. Tichuan, natural numbers, zero leh negative numbers zawng zawng chu 'the set of integers' (= nambar pum zawng zawng) kan ti a. Chu chu a hnuai ami ang hian, a tam dân indawtin, vei lam aṭanga ding lam zâwnga pung zêlin kan ziak tlar thei ang a; a chhinchhiah nan Z kan thlang a:

Z = {-∞, …, -n, ….. -3, -2, -1, 0, 1, 2, 3, ….. n, ….. ∞}

Heng nambarte hi rin (line)-a intlar angin, a tam dân indawtin "number line"-ah hetiang hian kan tlar thei ang:

← Negative Integers Positive Integers →

-∞, …-n, … -5, -4, -3, -2, -1, 0 +1, +2, +3, +4, +5, …+n, … +∞

Integer set Z-ah hian nambar zawng zawng a kim tawh ang em? Belh leh paih leh puntir atân chuan a kim a ang khawp mai. Integer 2 kan belh chuan, a belh chhuak (sum) chu integer bawk a ni tih a chiang mai. Paih (difference) pawh chuti bawk. Integer pahnih kan puntir pawhin a chhuak (product) chu integer bawk a ni. A sign (+ or -) positive or negative chungchângah chuan dân fel tak a awm a. Chu chu (1) sign inang product chu a positive (+) a; (2) sign inang lo product chu a negative (-) tih a ni mai a; kan hriat sa tho a ni.

Chuti chu nambar kan nei kim tawh a, kan fel tawh a ni mai em? Ni mai lo tlat! Sem (division) kan han thlen chiah hian a buaithlâk ta! Integer pakhatin integer dang kan sem chuan a chhuak (quotient) chu integer a ni vek ta lo tlat mai. A chuang a awm thei ta tlat mai. Entirnan: 6/3 = 2 a ni a; integer vek an la ni a. Mahse 37/8 = 4⅝ a ni a; integer, nambar pum a ni ta lo. He sem (division)-ah hian nambar pum 4(quotient) a chhuak a; mahse 8-a sem tham tawh loh a chuang 5 a la awm tlat mai, chu chu ⅝ tiin kan ziak ang; nambar pum tling lo, a phel - "hmun 8-a ṭhena hmun 5" tihna a ni, chu chu nambar pum tling lo '*fraction*' a ni. Integer pakhat integer dangin kan sem chuan integer dang leh a chuang/fraction chi hrang hrang a lo chhuak thei ta a; hei hi nambar chi dang a lo ni leh ta. Sem chhuah (quotient) tam tak chu integer a nih tho laiin a dang tam tak chu fraction emaw integer leh fraction inpawlh emaw a ni a. Han chhût vang vang ila, 'nambar pum' nge chhuak tam zâwk ang, 'fraction' emaw 'a pum leh a phel (fraction) inkawp' emaw? Chhiar sên loh ve ve a ni tih chu rilruah a chiang thei nghâl mai âwm e. Natural numbers zawng zawng chu chhiar sên loh (infinite) a ni a; integers zawng zawng pawh 'infinite' bawk – chuti chung chuan integer set chhûngah hian natural nambar set

chu a tel vek tho a ni. Mahse integer set-ah hian nambar a la kim chuang lo!

4. Rational numbers, Q

Integer set Z-a nambar pahnih insem chhuak, quotient zawng zawng chu, integer an nih vek loh avangin set Z chhûngah an leng lo va, chuvangin set Z aia huap zau zâwk, nambar set dang kan mamawh a ni. He set lian zâwk chhûngah hian set Z hi a la inkhung leng tih a chiang a. Chu set thar chu, nambar pum leh a them/fraction zawng zawng huap zo, 'the set of rational numbers' kan ti ang a; hetiang hian kan 'define' or 'represent' (hrilhfiah/entir) thei ang a, a chhinchhiah nan Q kan hmang ang:

The set of rational numbers,

$$Q = \{\text{all numbers of the form } p/q, \text{ where } p, q \in Z \text{ and } q \neq 0\}$$

Hei hi a 'algebra' ta deuh va, kan hrethiam em aw? "Rational nambar chu integer pahnih (p & q) insem chhuak zawng zawng a ni" tihna a ni mai. Integer zawng zawng pawh an la tel thei vek tih a lanna chu, q = 1 a nih chuan amah integer pângngai kha a ni tihna a ni mai. Hriat leh tûr chu p leh q hi eng integer pawh an ni thei tih hi a ni. Entirnan: p = 8, q = 4 ni se, p/q = 2; q hi 1 ni ta se p/q = 8 a ni ang a; q hi 5 ni ta se, p/q = (1 + 3/5) = 1.6 a ni ang. A nih leh q = 0 ni ta se, engtin nge ni ang? Nambar 0-a sem chuan awmzia a nei lo, 'undefined' kan ti a, a chhan chu 0-a kan sem chuan eng nambar pawh a chhuak thei vek tlat a ni; chuvangin 0-a sem chu phal loh, awmze nei lo, kan ti a; chu chu q ≠ 0 tiin, definition-ah khian, kan târ lang nghâl a ni e.

Tichuan, hetia kan han sawi mai theih chu "rational numbers" chuan natural nambar zawng zawng a huam a, whole nambar zawng zawng a huam a, integer zawng zawng a huam bawk a, a bakah fraction zawng zawng a huam vek bawk a ni. Hetiang hian ziak ila, a chiang mai âwm e:

N is a subset of W,
W is a subset of Z, and
Z is a subset of Q.

Kan zawhna ni ta chu - Q hian nambar awm thei zawng zawng a fûn kim tawh em? tih hi a ni. Kum tam tak chhûngin rational nambar bâk nambar dang a awm tih hriat a ni lo. Ngaihtuah thiam a ni lo va, chhût thiam pawh a ni lo. Tûnah erawh chuan hriat chian a ni vek tawh.

5. Irrational numbers

Rational number chu nambar pum pahnih insem chhuak zawng zawng a ni kan ti tawh a. Chu chu decimal-in a chhût chhuah a har lo va. Entirnan:

½ = 0.5 7/4 = (1 + ¾) = 1.75
22/7 = 3.14285714285714 .....

A hmasa berah khian decimal point/digit khat thleng chauh a awm a. A pahnihnaah chuan decimal point pahnih thleng a awm thung a. A pathumnaah hian decimal hi a tâwp mai lo va; mahse kan en chian chuan '142857' digit paruk hi, a indawt dân pângngaiin, a awm nawn a awm nawn mai a ni, chu chu '*recurring decimals*' kan tih kha a ni a. Chu chu a awm nawn ṭanna leh a tâwpna digit chungah chhunhan (dot) kan dah a, a fel mai a nih kha, kan hre reng âwm e. Hetiangin

1/7 = 0.14285714285714… = 0.142857
10/6 = 1.66666666…. = 1.6
233/144 = 1.6180555555 = 1.61805

Mathematician ho chuan rational nambar rêng rêng chu 'repeating/recurring' or 'terminating' decimal numbers/digits an ni zêl tih an hria a; chu chu rational number nihna pawimawh tak pawh a ni a..

Nambar, decimal tâwp thei lo (*non-terminating*), repeat/awm nawn bawk si lo (non-repeating/non-recurring) a awm chuan chu chu rational number a ni lo tih hriat a ni a. Chutiang nambar chu a awm rêng em? A tam mai - *irrational numbers* an ni. Kan thu-

puia kan han târlan pahnih te, φ leh π te khi, irrational nambar ve ve an ni a; hriat an nihna chu a rei tawh mai a; an chanchin hi a ngaihnawm êm êm a ni. Chu chu a ṭhen a zâr sawi kan han tum dâwn a ni a. A hmain nambar dang thu han sawi zawm leh lawk ila.

Tunah chuan irrational numbers chhiar sên loh a awm tih chu a chiang êm êm tawh a. Chung zinga kan han sawi mai theih chu √2, √3, √5 te hi a ni. Perfect square ni lo integer square root zawng zawng chu irrational nambar an ni ang a; heng lo pawh hi a dang tam tak a awm ang.

6. Real numbers, R

Rational numbers leh irrational numbers zawng zawng awm khâwm chu 'the set of real numbers' tih a ni ta a, a chhinchhiah nan R kan hmang a. Tichuan,

R = the set of real numbers consists of all rational and irrational numbers.

7. Imaginary number i and Complex numbers, C

Kan sawi lai hian han sawi kim law law ila. Real nambar kan tihah hi chuan nambar awm thei zawng zawng chu kim tawh âwm tak a ni; dik tak chuan kim ang pawhin a sawi theih tho mai. Number line kan tih khi chuan real numbers zawng zawng chu a fûn kim tawh a. Mahse 'imaginary number' a la awm ta fo va! Chu chu

$\sqrt{-1} = i$

hi a ni. Mathematics dânah chuan nambar square rêng rêng chu positive nambar a ni tur a ni. A chhan chu negative nambar ve ve a inpuntir chuan, sign inang inpuntir chu a + lo thei lo. Mahse kan hmuh ang khian $i^2 = ixi = (\sqrt{-1})x(\sqrt{-1}) = -1$. Chuvangin, *i* hi real number a ni thei lo, imaginary number kan ti lo thei lo a ni. Tichuan, number chi thar kan neih leh tak chu "complex numbers" a ni a; tichuan the set of complex numbers is

C = {all numbers of the form a + *i*b where a, b € R}

Complex numbers chu number line-ah an leng ve tawh lo va, number plane a ngai tawh a ni. Sawi chian tûr a la awm nual nâ a, duh tâwk tawh mai ang. Kan sawi tum ber a ni chiah lo bawk si nên. Tunah chuan kan thupuiah lût dâwn tawh ila.

π (PI) CHANCHIN TLEM

π (pi) hi chu kan hre tlângpui âwm e. Circle circumference a diameter-a sem a ni mai a, hetiangin

π = Circumference of a circle/Its diameter
= C / d
= 3.14159 26535 ….

Hei hi mâwl tê angin a lang nâ a, decimal tâwp thei lo chi, awm nawn bawk si lo - *irrational number* a ni mauh mai! Hetiang nambar chungchâng chu a chung lamah kan sawi nual tawh kha. Tûna kan han ziak chhuah khi decimal point/digit sâwm thleng a ni.

π hi a pawimawh êm êm a, circle nêna inzawmna nei miah lo thil tam takah a lo lang fo mai a ni. A tel lo chuan mathematics hi a kalpui hleih theih loh a ni ber mai. Chuvangin chanchin pawh a ngah tawh êm êm, a ṭhen a zâr chauh kan sawi thei ang a.

Mnemonic

π (pi) = 3.14159 26535….. nihna (value) dik tak chu ziak chhuah sên pawh a ni lo va; hriat reng awlsam nan thu phuah tam tak siam a ni tawh (nambar hriat reng awlsam nana thuphuah = mnemonic). College kan luh ve tirh a, kan zirtirtu min hrilh chu ka la hre reng a, mi dang tam tak ka hrilh ve tawh nghe nghe. Chu chu:

π kan hriat reng theihna tur thu (sentence) pakhat chu, Sapṭawngin hei hi a ni:

Yes, I have a number

He thu (sentence)-a hawrawp awm zât zêl hi kan chhiar chuan 'Yes' ah 3, 'I' ah 1, 'have' ah 4, 'a' ah 1, 'number' ah 6; tichuan 3.1416 a lo ni ta a. He nambar hi a chunga kan târ lan nên khian kan entawn chuan π (pi) "correct to 4 decimal places" a ni a.

Mizo ṭawnga ka'n phuah ve chhin chu :

"Nia, a nasa a ropui maktinkim an riruat
3 . 1 4 1 5 9 2 6
zozai kan enpui"
5 3 5

Hei hian a chunga π value kan târlan zawng khi, 10 decimal places thleng, a lam chhuak kim a ni. Chiartuten han phuah chhuak ve chhin teh u, a ngaihnawm duh ngawt ang.

Tun hnaia magazine pakhata ka chhiar chu hei hi a ni:

"How I want a drink, alcoholic of course,
3 . 1 4 1 5 9 2 6
after the heavy lectures involving
5 3 5 8 9
quantum mechanics"
7 9

(correct to 14 decimal places!). Hetiang atân hian intihsiakna kan han buatsaih dâwn êm ni? Rilru sêng peih chuan thu han phuah belh zel teh u.

Fraction pângngai (rational number) π tlukpui atana an hman uar deuh ber chu 22/7 hi a ni. Kan duhtuiin kan duh uluk hle a, 'high accuracy' kan phût a nih loh chuan, hei hi π aiawh turin a pawm tlâk hle rêng a (approximation atân a ṭha tâwk viau a ni). High school velah chuan 22/7 hi kan hmang nasa a, ṭhenkhat chuan he nambar hi π tlukpui (value) dik tak emaw kan lo ti mai ang tih a hlauhawm thei hial âwm e; a chûlchamna (approximation) chauh a ni tih i hre reng tawh ang u.

$$22/7 = 3.14285\ 71428\ 57132\ \ldots$$

hi kan han en ngun chuan decimal-ah hian 142857 hi recurring a ni a. Decimal point/digit hnih thleng π hi a tluk avangin duhtui vak lo tân chuan hman âwm tak pawh a ni rêng a ni han ti leh hrâm ila. He nambar 22/7 hi hriat lâr a ni a, tam tak chuan π entirna anga ngaiin "22 July" (= 22/7) ni hi kum tin he nambar hriat chhuah ni atân an hmang a, "Pi Approxmination Day" an vuah hial a ni.

Chu lovah chuan π digit hmasa pathum 3.14 hi 'March 14' nen an sawi zawm bawk a, he ni hi "Global Pi Day" tiin, chhang bial engemaw bikte urin, ei tûr tuihnai buatsaihin, an lâwm ṭhin an ti bawk. Thil mak leh chhinchhiah atâna ṭha ve mai mai tak pakhat chu, March 14 hi physicist ropui ber zinga mi Albert Einstein-a pian ni a ni.

BC 429 lam daiha piang, Chinese mathematician pakhat, Zu Chongzhi chuan π chu 355/113 = 3.14159292… niin a chhût chhuak a (decimal point/digit paruk thleng a dik a). Hei hi kum 1000 chhûng lai a dik ber anga pawm a ni. AD 1579 kumah a aia dik zâwk chhût chhuah a ni chauh a. He mi kum hian French mathematician pakhatin digit 9 thlengin a rawn chhût dik a, 1610-ah digit 35 thleng. Tin, 2010-ah phei chuan he nambar π hi 5 trillion digits thleng chhût chhuah a ni tawh. A chhût chhuaktute chu Japanese system engineer, Shigeru Kondo leh American computer science zirlai, Alexander Yee an ni a; ni 90 chhûng an bei a nih chu!

Music lamah pawh π (pi) hian hnuhma a nei nasa hle. A puala hla phuah leh album lâr tak takte pawh a awm an ti a. Chhinchhiah tlâk tak pakhat chu, musician nu pakhat, Kate Bush-i chuan, 2005 kum khan he number-a digit indawt hi hlain a sa a, digit 100 thleng a sa a. A ngaithlatu zinga lo chik ngun peih deuhte erawh chuan digit 50 vêl thleng chuan a tidik a, a bâk chu a phuahchawp zawm deuh mai mai a ni an ti!

Pi kan kalsan hmaa han târlan mai mai

atâna ṭhaa ka hriat chu

π= Circumference/diameter = C/d = C/2r

where C is circumference, d is diameter and r is radius of the circle.

Hence, we have C = 2πr.

Also, area of the circle, A = πr²

Φ (PHI) CHANCHIN VE THUNG

Hei hi Euclid-a te hun lai pawha hriat tawh a ni a, chanchin a ngah tawh êm êm a, kan sawi sêng dâwn lo va. Eng nge a nih chiah tih aṭangin han ṭan phawt ila. A hming bul ber leh lar ber chu Golden Section = Golden Ratio a ni. Ṭhenkhat chuan 'Divine Ratio' an ti a. Golden Number leh Divine Proportion ti te pawhin sawi a ni bawk.

A lem nên sawi fiah han tum ila :

Golden Section

_____x_____._____1_____
A C B

Golden section chu hetiang hian sawi a ni: *"A chunga rin (line) AB khi C-ah khian tan ta ila, ratio AB/AC leh AC/CB te hi an intluk chuan 'golden section' a ni"* tiin.

Hei hi a ho hlein a lang nâ a, he ratio, Golden Ratio, tih hi namên lovin a pawimawh a, rin lohna lam leh beisei lohna lam tak takah a rawn inlâr fo ta mai a. Chuvangin, mi ṭhenkhat phei chuan *Divine Ratio*, *Divine Number* (= Pathian Nambar) an vuah hial a.

Han chawk chhin ta ila:

AB/AC = AC/CB

Tah hian AB = x + 1, AC = x; CB = 1

Tichuan x + /x = x/1

Cross-multiplication hnua, equation lo chhuak ta chu

$x + 1 = x^2$

Chu chu $x^2 - x - 1 = 0$ a ni. Hei hi quadratic equation a ni a, a root-te chu

$x = (1 + \sqrt{5})/2$

leh $x = (1 - \sqrt{5})/2$ te.

A chung zâwk khi kan chhût chhuah chuan x = 1.61803 39887 … kan hmu a; "x hi Golden Ratio chu a ni ta a ni". A decimal point/digit hi 'non-repeating' leh 'non-terminating' a ni a; chuvangin x hi *irrational number* a ni.

He nambar hi a danglam riau mai, hetiangin:

A square chu $x^2 = 2.61803\ 39887\ldots = x + 1$

A reciprocal chu $1/x = 0.61803\ 39887\ldots$
$= x - 1$

Chu bâkah, a root pahnihna khi = $(1 - \sqrt{5})/2 = -0.61803\ 39887\ldots = -1/x$

Hetiang nambar hi a dang a awm lo rêng rêng!

Kum zabi sawmhnihna bul lamah khân mathematician Mark Barr-a chuan BC 490-430 lam daih tawh a, Greek mi, milim chher thiam (great Greek sculptor) Phidias-a hming chawiin, Golden Ratio hi a chhinchhiah nan Greek hawrawp φ (phi) hi a lo thlang a; chu chu khawvêl pumah hman zui reng a ni ta a. Kan chhût chhuahnaa x khi a ni.

Nambar π chungchânga kan sawi lan tawh, hriat reng awlsam nâna thu phuah 'mnemonic' kan tih kha φ tân pawh hian ka han phuah chhin ve a, chu chu

φ (phi) = 1.61803 39887 …..

"A! riruat a mak-in-mak o kan hre
1. 6 1 8 0 3 3
thei-bil-lo in-rin-aia mak-zo-zai kimbiai"
9 8 8 7
(correct to 10 decimal places!)

tih hi a ni. Chhiartuten a dang han phuah chhuak zawm teh u le!

Tûn hnaia lehkhabu ka chhiar pakhat "*The Golden Ratio - the Story of phi, the World's Most Astonishing Number*" (2002 Broadway Books) Mario Livio ziakah chuan he nambar 'phi' hi decimal place/digits 2000 thleng, a bu phêk 81 leh 82-ah a rawn ziak chhuak tuar mai a. Heta ka han ziak ve hi chu digit sâwm thleng chauh a ni a, hei hi zirtu tam ber tih dân tlângpui a ni - decimal point/digit sâwm thleng chauhva ziak chhuah hi.

A bul leh chhût chhuak hmasatu

Hetiang nambar, decimal tâwp thei lo, digit awm nawn (recurring) bawk si lo, hian hman lai aṭang tawhin mi rilru a tibuai a, mak tih a hlawh tawh rêng a ni an ti a. Sawi dân pakhatah chuan, BC kum zabi ngana daih tawhah khan Grik mathematician pakhat, Hippasus of Metapontum chuan Golden Ratio hi nambar pum (whole number) ni si lo, fraction (rational number) pawh ni bawk si lo, decimal tâwp thei lo nambar a ni tih a han hmuh chhuah chuan mathematician hmingthang leh ropui ber zinga mi, Pythagoras-a hnung zuitute chu mak tiin a ṭhâwng nasa êm êm a ni an ti. Chutiang nambar danglam an han hmu/hre chhuak chuan an rilru a khawih nasa a, an nun pumpui a nghawng thûk tak meuh a ni. Ṭhenkhat sawi dân phei chuan Pythagorean-ho chuan mak an tih lutuk avangin bâwngpa zâ an talh/hlan (sacrifice) hial niin an sawi.

Hetiang nambar, a chung lam pawha kan sawi lan tâk, irrational/incommensurable numbers, hmuh chhuah (hriat chhuah) a nih hun hi chiang takin a hriat theih tawh loh naa, zir bingtu tam ber chuan BC kum zabi ngana vêl ni ngeiin an ring. A tir chuan, tam takin pawm harsa an ti a, an dodal nasa a, thil thiang lovah an puh hial a, a chhût chhuak hmasatu pawh an tuithlâr hial a ni tein sawi a ni bawk.

Khawvêl hun liam tawh zawng zawng han thlir kir pawhin, mathematician ropui leh lâr zawng zawngte chu - hman laia Grik mi ropui leh lar ber ber Pythgoras leh Euclid te aṭangin, khawvêl hun laihawl vêla Italian mathematician chhuanvâwr, Leonardo of Pisa te, Renaissance huna astronomer hmingthang ber Johannes Kepler te leh tûn hnai lama Oxford physicist chhuanvawr Roger Penrose te thlengin - he nambar mak tak buaipuitu leh a ze hrang hrang chhui leh chhût vêla hmanhlel an ni hlawm vek mai. Chu achhapah, φ (Golden Ratio) buaipuitute chu mathematician chang an ni lo va, biologists, artists, musicians, historians, architects, psychologists leh rilru leh thlarau lam ram luhchilh mite thlengin he nambar danglam tak hi an zir a, an ngaihven a, hman ṭangkai an tum hlawm. Heti khawpa nambar mi tinrêng ngaihven leh buaipui hlawh hi mathematics lo ṭhan chhohnaah hian a la awm ngai lo rêng rêng a ni.

φ (phi) chanchin kan tihtâwp hmain, a chunga hming kan han târlan zinga mi, Leonardo of Pisa hi, hmêlhriat bulfûk deuh i han tum lawk teh ang. Ani hi a hming tak chu Leonardo Fibonacci (1170s-1240s) a ni a, 1170 chho vêla piang nia hriat a ni. Lehkhabu a ziak hmasak ber, lâr zui ta êm êm chu *Liber abaci* (Book of the abacus) tih a ni a, 1202 kuma chhuah a ni. Ani hming kan han lam chhuah duhna chhan ber chu *Fibonacci sequence* leh φ (phi) a inkungkaih hnaih êm êm vâng a ni a. Kan chhût theih chinah engmah inzawmna an neih hriat a ni si lo. He sequence lo chhuah dân pawh hi thil dang daih aṭangin a ni a, tûnah chuan kan sawi sêng lo ve. Fibonacci sequence chu

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ….

hi a ni. A nambar kal zêl dên chu - "a dawt nambar tur chu a hma nambar pahnih belh zêl a ni" tih hi a ni mai. Han en ila, 13 dawttu chu 21 = 13 + 8; chutiang zelin 144 dawta nambar lo awm leh tur chu 144 + 89 = 233; tichuan 233 dawttu tur chu 233 + 144 = 377 a ni ang. Chutiang zêl chuan a kal ang. Fibo-

nacci sequence tih hming phuahtu tak phei chu French mathematician, Édouard Lucas (1842-1891) a ni.

"Fibonacci sequence-a nambar/term eng-pawh chhût chhuak tûr chuan, a formula chu

$$Fn + 2 = Fn + 1 + Fn, \text{ where } n \in N$$ a ni"

tih hian chhiartute a hrilhfiah vek mai em aw? A formula-ah khian n = 2 a nih chuan F4 = F3 + F2 → 3 = 2 + 1. Chutiang zêlin, n = 5 a nih chuan Fn+2 chu F7 a ni ang a, chu chu sequence kan ziak tlar a 7-na a ni a 13 a ni. Chutiang zelin Fn+1 chu F6 a ni ang, chu chu 6-na, 8 a ni. Fn = F5 = 5. Tichuan F7 = F6 + F5 → 13 = 8 + 5; a dik a.

Fibonacci sequence leh φ (phi) a inkung-kaihna chu "sequence-a nambarte, a hmaa mi hian a dawt chiah sem ta ila, chu chuan golden ratio a hnaih chho tial tial a ni" tih hi. Hei hi thil mak tak leh chhinchhiah tlâk êm êm a ni. A ṭhen a zâr han entir ila -

89/55 = 1.61818 18181… › φ
144/89 = 1.61797 75280… ‹ φ
233/144 = 1.61805 55555… › φ
377/233 = 1.61802 57510…. ‹ φ

Kan sem chhuah khi kan en chuan, pali zingah khian 1-na leh 3-na khi φ aiin an tam a; 2-na leh 4-na te khi φ aiin an tlêm thung a. A lan dân chuan, hetiang hian sem chhun-zawm zêl ta ila, a inhnaih chho tial tial ang a, φ (phi) nên hian a intluk teuh tial tial dâwn a ni. Hei hi mathematical notation fel tak hmangin hetiang hian ziak theih a ni:

$$\frac{Fn+1}{Fn} \rightarrow \varphi \text{ when } n \rightarrow \infty$$

He inkungkaihna, 1611 kuma hmuch-huaktu chu, a hmaa a hming kan lam tawh hnu, Johannes Kepler, German astronomer hmingthang tak bawk kha a ni tih pawm a ni a; mahse tunge a nih hriat chhuah theih tawh loh Italian mathematician-in a hma hian a lo hre tawh nia sawi an awm bawk.

Tûn ṭumah hian phi chanchin hi pure mathematics lam thlirna aṭangin kan sawi tam ber a. Kawng dang tam takah a lang chhuak nasa êm êm tho va. Painting lâr leh hmingthang tak taka a tel vena sawi tur tam tak a awm a. Chu bakah, Five Platonic Solids tia hriat lâr - tetrahedron, cube, octahedron, dodecahedron leh icosahedron - hengho leh Golden Ratio (phi) inzawm vêl dân leh inkungkaih kual dân ringawt pawh hi a zir mi leh a tuipui mite tân chuan sawi tur tam tak, mak tak tak, ngaihnawm tak tak, chhinchhiah tlâk tak tak a kuh ṭul mai. Chung chu kan luhchilh tawh rih lo ang a.

Chu bakah Fibonacci sequence chanchin leh φ (phi) chanchin te, an inkungkaih tawn dân te pawh sawi tur tam tak a la awm ang; chung chu tûn atân chuan chhui zêl lovin du-htâwk tawh rih mai ang. A remchân chuan kan la sawi zêl ang chu.

