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Digit sums of whole numbers – an analysis

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INTRODUCTION

The Groundwork

Every morning on my way to office, I had to drive through the whole Aizawl city from Mission Veng to Chaltlang. I could not help looking at the registration numbers of cars in front or of those parked on the roadsides, such as, for example, MZ 01 D 2143, MZ 01 E 8379, MZ 01 F 5617, etc. All car numbers are four digit whole numbers. As a man of mathematics, I often add up the digits in my mind, noting the various sums of the digit numbers *viz.* 10, 27, 19 (for the above 3 car numbers). A thought came to me that the lowest possible sum for a 4-digit number would be 1 corresponding to which all possible registration numbers that can be allotted are 4 (four), *viz.* 0001, 0010, 0100 and 1000. It is also clear that for a given alphabet, all possible car registration numbers that can be allotted would be 9999 altogether, *viz.* MZ 01 F 0001 … F 9999. The next car after F 9999 will have to be allotted G 0001 and so on.

The Question and Questions

Now, the question that poses itself is: How many cars will have the same sum or total of the 4 digits of their registration numbers? How are these 'numbers' distributed statistically? Is it possible to classify them in some mathematical order? Is there an underlying formula for analysis of these various digit sums for ease of calculations? Can we deduce a general formula or guidelines to deal with digit sums of whole numbers of n digits, where n stands for any positive integer?

Some Simple Observations

It is easy to see that the digit sums of all 4-digit whole numbers will range from 1 to 36, the largest number being 9999 for which the sum of the digits is 9+9+9+9 = 36. I hope this is obvious to everyone. At this point, let me state that, for the sake of completeness and in order to facilitate proper analysis mathematically, it is desirable to include 0 also as 0000 in the set of all 4-digit whole numbers. Hence, if S is the set of all 4-digit whole numbers, then

S = {0, 1, 2, …, 9998, 9999}, so that the cardinal number of set S = 10,000.

Comments: It is perhaps important to clarify that the set S above contains all whole numbers from single digit to 4 digits. This is acceptable and logically and mathematically sound as the number 1, for example, can be

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regarded as 1, 01, 001 or 0001 without changing its value. It is also clear that there is only one number of 4-digit whose digit sum is 0, *viz.* 0 itself; and 0 = 0000; and also only one 4-digit number whose digit sum is 36, *viz.* 9999! Further, it can be deduced that there can be only four 4-digit whole numbers whose digit sums will be 1, *viz.* 0001, 0010, 0100, 1000; more properly 1, 10, 100 and 1000. And so on.

The First Question Restated

The original question raised above may now be rephrased in a mathematical language as: How many 4-digit numbers are there whose digit sum (DS) will be s, where s can take values from 0,1,2,3, … 34,35,36?

THE APPROACH

Step by Step

From the point of view of mathematical analysis, there is no valid reason to confine our enquiry and research to the 4-digit numbers only and no logical basis to begin with 4-digit numbers. Instead, our approach should be to start from the beginning which, in our case, must be 1-digit or single digit numbers; and then to go on to 2-digit, 3-digit numbers and so on to higher digit numbers. Therefore, we should adopt this approach in keeping with the principle of mathematical analysis.

ONE-DIGIT WHOLE NUMBERS

The Case of 1-Digit Whole Numbers

This may be regarded as the trivial case. It is simple and straight forward. All the 1-digit whole numbers are 0,1,2,3,4,5,6,7,8,9. It is easy to verify that there are 10 1-digit whole numbers. Further, it is obvious that all possible digit sums range from 0 to 9. In fact, there is no question of taking the sum of the digits as there is only one digit; so this is the trivial case. The analysis is clearly displayed in Table 1.

Table 1.

Digit sum (DS)	Actual number	Possible Permutation	Total number having the sum
0	0	Nil	1
1	1	Nil	1
2	2	Nil	1
3	3	Nil	1
4	4	Nil	1
5	5	Nil	1
6	6	Nil	1
7	7	Nil	1
8	8	Nil	1
9	9	Nil	1
	Total		10

TWO-DIGIT WHOLE NUMBERS

The Case of 2-Digit Whole Numbers

Let us denote the set of all whole numbers of two digits by S(2). Then

S(2) = {00, 01, 02, 03 …10,11 …98, 99} = [10x + y, where x, y € {0,1,2,…,9}]

General Comment: It must be clarified at this point that we must include all single digit numbers also in the set of 2-digit numbers because the number 1 may be regarded as 01 without altering it value, 2 as 02, 3 as 03, and so on. This is necessary and also mathematically sound and proper. Indeed, the same approach will be adopted throughout in our analysis, so that 1 = 01, 001, 0001, … and will be legitimately included in the set of whole numbers of any number of digits. This must be understood very clearly by all concerned. Therefore, it is clear that S(2) is a finite set of 100 elements. It is also obvious that the digit sums of all 2-digit whole numbers range from 0 to 18 as 9+9 = 18.

Preliminary Analysis

Let us take the case of 2-digit numbers having digit sum of 5, for example. Then possible pairs are 0,5; 1,4; and 2,3. Does it mean that there are only 3 numbers of 2-digits having 5 as digit sum? No, these 3 pairs are only the possible *combinations*. We have to consider their possible *permutations* as well. In the case of 1-digit numbers considered above, there is no question of permutation as all the numbers consist of single digit only. So, it is easy to see that the actual 2-digit numbers having digit sum 5 are 05 = 5, 50, 14, 41, 23 and 32; altogether 6 different numbers. We can reassure ourselves that this is the correct and complete answer.

Complete Analysis

We have to take the cases of all possible digit sums for 2-digit numbers *viz*. 0, 1, 2, 3, … 16, 17, 18. The calculations are straight forward and can be done with a bit of patience by anyone with normal computation skills. The complete analysis of this case has been done and given below in a tabular form.

THREE-DIGIT WHOLE NUMBERS

The Case of 3-Digit Whole Numbers

Let us denote the set of all whole numbers of three digits by S(3). Then

S(3) = {0, 1, 2, …100,101, …998, 999} = [100x + 10y + z, where x, y, z € {0,1,2,…,9}]

It is easy to see that S(3) is a finite set of 10,000 elements. It is clear that the digit sums

Table 2.

Digit sum (DS)	Actual numbers having the sum, DS	Possible Combinations	Permutations : actual numbers having corresponding sum, DS (Frequencies denoted by F)	Total (F)
0	00	0,0	00	1
1	01, 10	0,1	01, 10	2
2	02, 20, 11	0,2; 1,1	02,20,11	3
3	03, 30, etc.	0,3; 1,2	03,30,12,21	4
4	04, 40,22, etc.	0,4; 1,3; 2,2	04, 40, 13,31,22	5
5	05, 50, 14, etc.	0,5; 1,4; 2,3	05,50,14,41,23,32	6
6	06,60.24, etc.	0.6;1,5; etc.	06,60,15,51,24,42,33	7
7	07, 70, 16, etc.	0,7; 1,6; etc.	07,70,16,61,25,52,34,43	8
8	08, 80, 26, etc.	0,8; 1,7; etc.	08,80,17,71,26,62,35,53,44	9
9	09, 90, 45, etc.	0,9; 1,8; etc.	09,90,18,81,27,72,36,63,45,54	10
10	19, 91, 28, etc.	1,9; 2,8; etc.	19,91,28,82,37,73,46,64,55	9
11	29, 92, 47, etc	2,9; 9,2; etc.	29,92,38,83,47,74,56,65	8
12	39, 93, 66, etc.	3,9; 9,3; etc.	39,93,48,84,57,75,66	7
13	49, 94, 58, etc.	4,9; 9,4; etc.	49,94,58,85,67,76	6
14	59, 95, 86, etc.	5,9; 9,5; etc.	59,95,68,86,77	5
15	69, 96, 78, etc.	6,9; 9,6; etc.	69,96,78,87	4
16	79, 97, 88	7,9; 9,7; etc.	79,97,88	3
17	89, 98	8,9; etc.	89,98	2
18	99	9,9	99	1
			Total	**100**

Table 3.

Digit sums (DS)	Possible Combinations, C	Possible Permutations, P Total of all permutations = F (Frequencies)	F	dF
0	0,0	000	1	
1	0,1	001,010,100	3	2
2	0,2;0,1,1	002,020,200,011,101,110	6	3
3	0,3;0,1,2;1,1,1	3 + 6 + 1	10	4
4	0,4;0,1,3;0,2,2;1,1,2	3 + 6 + 3 + 3	15	5
5	0,5;0,1,4;0,2,3;1,1,3;1,2,2	3 + 6 + 6 + 3 + 3	21	6
6	0,6;0,1,5;0,2,4;0,3,3;1,1,4; 1,2,3;2,2,2	3+6+6+3+3+6+1	28	7
7	0,7;0,1,6;0,2,5;0,3,4;1,1,5; 1,2,4;1,3,3;2,2,3	3+6+6+6+3+6+3+3	36	8
8	0,8;0,1,7;0,2,6;0,3,5;0,4,4; 1,1,6;1,2,5;1,3,4;2,2,4;2,3,3	3+6+6+6+3+3+6+6+3+3	45	9
9	0,9;0,1,8;0,2,7;0,3,6;0,4,5; 1,1,7;1,2,6;1,3,5;1,4,4;2,2,5; 2,3,4;3,3,3	3+6+6+6+6+3+6+6+3+3+6+1	55	10
10	0,1,9;0,2,8;0,3,7;0,4,6;0,5,5; 1,1,8;1,2,7;1,3,6;1,4,5;2,2,6; 2,3,5;2,4,4;3,3,4	6+6+6+6+3+3+6+6+6+3+6+3+3	63	8
11	0,2,9;0,3,8;0,4,7;0,5,6;1,1,9; 1,2,8;1,3,7;1,4,6;1,5,5;2,2,7; 2,3,6;2,4,5;3,3,5;3,4,4	6+6+6+6+3+6+6+6+3+3+6+6+3+3	69	6
12	0,3,9;0,4,8;0,5,7;0,6,6;1,2,9; 1,3,8;1,4,7;1,5,6;2,2,8;2,3,7; 2,4,6;2,5,5;3,3,6;3,4,5;4,4,4	6+6+6+3+6+6+6+6+3+6+6+3+3+6+1	73	4
13	0,4,9;0,5,8;0,6,7;1,3,9;1,4,8; 1,5,7;1,6,6;2,2,9;2,3,8;2,4,7; 2,5,6;3,3,7;3,4,6;3,5,5;4,4,5	6+6+6+6+6+6+3+3+6+6+6+3+6+3+3	75	2
14	0,5,9;0,6,8;0,7,7;1,4,9;1,5,8; 1,6,7;2,3,9;2,4,8;2,5,7;2,6,6; 3,3,8;3,4,7;3,5,6;4,4,6;4,5,5	6+6+3+6+6+6+6+6+6+3+3+6+6+3+3	75	0
15	0,6,9;0,7,8;1,5,9;1,6,8;1,7,7; 2,4,9;2,5,8;2,6,7;3,3,9;3,4,8; 3,5,7;3,6,6;4,4,7;4,5,6;5,5,5	6+6+6+6+3+6+6+6+3+6+6+3+3+6+1	73	-2
16	0,7,9;0,8,8;1,6,9;1,7,8;2,5,9; 2,6,8;2,7,7;3,4,9;3,5,8;3,6,7; 4,4,8;4,5,7;4,6,6;5,5,6;	6+3+6+6+6+6+3+6+6+6+3+6+3+3	69	-4
17	0,8,9;1,7,9;1,8,8;2,6,9;2,7,8; 3,5,9;3,6,8;3,7,7;4,4,9;4,5,8; 4,6,7;5,5,7;5,6,6	6+6+3+6+6+6+6+3+3+6+6+3+3	63	-6
18	0,9,9;1,8,9;2,7,9;2,8,8;3,6,9; 3,7,8;4,5,9;4,6,8;4,7,7;5,5,8; 5,6,7;6,6,6	3+6+6+3+6+6+6+6+3+3+6+1	55	-8

Table 3 (continued).

19	1,9,9;2,8,9;3,7,9;3,8,8;4,6,9; 4,7,8;5,5,9;5,6,8;5,7,7;6,6,7	3+6+6+3+6+6+3+6+3+3	45	-10
20	2,9,9;3,8,9;4,7,9;4,8,8;5,6,9; 5,7,8;6,6,8;6,7,7	3+6+6+3+6+6+3+3	36	-9
21	3,9,9;4,8,9;5,7,9;5,8,8;6,6,9; 6,7,8;7,7,7	3+6+6+3+3+6+1	28	-8
22	4,9,9;5,8,9;6,7,9;6,8,8;7,7,8	3+6+6+3+3	21	-7
23	5,9,9;6,8,9;7,7,9;7,8,8	3+6+3+3	15	-6
24	6,9,9;7,8,9;8,8,8	3+6+1	10	-5
25	7,9,9;8,8,9	3+3	6	-4
26	8,9,9	3	3	-3
27	9,9,9	1	1	-2
		Total numbers	**1000**	

for all 3-digit whole numbers will range from 0 to 27 as 9+9+9 = 27.

Let us consider the case of 3-digit numbers whose digit sum is, for example, 4. The possible combinations are 0,0,4; 0,1,3; 0,2,2; and 1,1,2 – altogether 4 different combinations. The possible permutations for 0,0,4 combination are 004, 040, and 400; hence, there are 3 numbers in S(3) whose digit sum is 4, *viz*. 4, 40 and 400. But these are not the only ones. In fact, the next combination 0,1,3 can permute among themselves in 6 ways; the third 0,2,2 in 3 ways; and finally 1,1,2 also in 3 ways. So, adding up all these permutations, we have 3+6+3+3 = 15. Therefore, there are altogether 15 three-digit whole numbers whose digit sum is 4.

So, in order to complete the 3-digit case analysis, we will have to find all the possible numbers of combinations and permutations of 3-digit whole numbers corresponding to all the possible digit sums from 0 to 27.

At this point, it is desirable to introduce the tools/symbols for the general analysis that will be applicable to any set of whole numbers of n-digits where n takes any integral values such as 3, 4, 5, … and so on. We have already dealt with cases where n is either 1 or 2. Let us use C and P for possible combinations and permutations of the digit contents; and G (groups) and F (frequencies), for the total numbers respectively of combinations and permutations of n-digit whole numbers having a particular digit sum, DS under consideration.

It follows that for a 3-digit whole numbers there can be *3 possible different kinds of combinations, viz.* all alike such as 111, 222, etc. which may be denoted by C1; one different and two alike such as 011, 244, etc which may be denoted by C2; and lastly, all three different such as 012, 478, etc which may be denoted by C3. It is easy to see that the permutations P1, P2 and P3 corresponding to C1, C2 and C3 are 1, 3 and 6, respectively.

We may now restate the example considered above regarding digit sum of DS = 4 for which the combinations are 0,0,4; 0,1,3;,0,2,2 and 1,1,2 as already pointed out above. These are combinations of C2, C3, C2 and C2 kinds respectively. Hence, we have 3C2 and C3 leading to G = (3+1) = 4, corresponding to which we have P2, P3, P2 and P2. So we have 3P2 + P3 = 3x3 + 6 = 9 + 6 = 15 = F; so that the number, G (groups) of possible combinations and F (frequencies) of possible permutations of 3-digit whole numbers corresponding to the digit sum DS = 4 are G = 4 and F = 15, respectively (see Table 3).

Just for interest, let us take DS = 21. The possible combinations are 7,7,7 = C1; 6,7,8 = C3; 5,7,9 = C3; 4,8,9 = C3; 3,9,9 = C2; 6,6,9 = C2; 5,8,8 = C2. Hence, we have C1 + 3C2 + 3C3 = 7 = G corresponding to which we have P1 + 3P2 + 3P3 = 1 + 3x3 + 3x6 = 1 + 9 + 18 = 28 = F.

It would need a lot of patience and care to ensure that all the possible combinations have been identified; once that is done, however, it would be easy to calculate G, the total possible combinations and then F, the total possible corresponding permutations. There is, of course, a systematic way to ensure that all possible combinations are found correctly. The Table 3 below gives the complete picture of the full analysis of the 3-digit case without details of C and P while Table 4 gives these details as well.

Observations

The distribution of the frequencies, F (or clusters) of 3-digit numbers in relation to their digit sums DS is symmetrical. This is apparent from the column headed by (F) in the above Table 3.

This is also true in the case of 2-digit and 4-digit numbers as can be seen from Table 2 and Table 5, respectively..

The maximum frequency, *viz.* 75 occurs twice against DS = 13 and DS = 14 in Table 3. The maximum DS in this case is 27 and half of 27 is 13.5. It is significant that 3-digit number clusters are found with those having DS 13 or 14 which are midway between 0 and 27, the minimum and maximum possible DS.

Also from Table 3, it may be noted that the maximum F *viz.* 10 occurs against DS = 9 which is exactly half of 18, the maximum possible DS in this case. So we find that the distribution of F is symmetrical about the mid-point of DS, which in this case is 9 = 50 % of 18 (maximum DS). Similarly, both G and F are distri-buted symmetrically about DS = 18 (50% of 36) in the 4-digit case(*cf.* Table 5).

It follows that the centre of symmetry in the case of 3-digit numbers may also be regarded as being located at 13.5 = 50 % of 27 (maximum DS).

It is apparent that this pattern will be one of the fundamental properties of the subject under this particular investigation/research (*cf.* Tables 2, 4 & 5).

Lastly, the last column in Table 3 is headed by dF which means the first degree difference between the succeeding values of F. Here again we may observe an interesting pattern. More attention may be given to this aspect of our analysis by observing higher degrees of differences, such as ddF, dddF, and so on.

Table 3 Recasted

Using our adopted symbols, it will be useful and worthwhile to recast the above Table 3. This has been done with care and given below as Table 4. Both Table 3 and 4 are, of course, the same for all practical purposes.

FOUR-DIGIT WHOLE NUMBERS

The Case of 4-Digit Whole Numbers

We are now in a position to consider our original question relating to the digit sums of 4-digit whole numbers.

Let us denote the set of all whole numbers of four digits by S(4). Then

S(4) = {0, 1, 2, 3, …., 9998, 9999}

= [Numbers of the form 1000x + 100y + 10z + w, where x,y,z,w €{0,1,2,…, 8, 9}]

It is clear that S(4) is a finite set of 10,000 elements. We can now proceed with our analysis of this case by using our adopted symbols.

It is easy to see that there can be *5 different kinds of combinations for the four digits of the 4-digit whole numbers*. So, let C0 denote possible combinations for all 4 same digits such as 1111, 3333, etc; C1 for 1 different and 3 same such as 1112, 4777, etc; C2 for 2 different and 2 same such as 2233, 5588, etc; C3 for 3 dif-

Table 4.

Digit sums (DS)	Possible Combinations, C with total = G	Possible Permutations, P with total = F	F	dF
0	C1(0,0) = **1**	P1 = **1**	1	
1	C2(0,1) = **1**	P2 = **3**	3	2
2	C2(0,2); C2(0,1,1) = 2C2 = **2**	2P2 = 2x3 = **6**	6	3
3	C2(0,3);C3(0,1,2);C1(1,1,1)=**3**	P2 + P3 + P1=3+6 +1 =**10**	10	4
4	C2(0,4);C3(0,1,3);C2(0,2,2)C2(1,1,2) = 3C2 + C3 = **4**	3P2 + P3 = 3x3 + 6 = 9 + 6 = **15**	15	5
5	C2(0,5);C3(0,1,4);C3(0,2,3)C2(1,1,3);C2(1,2,2) =3C2+2C3= **5**	3P2 + 2P3 = 3x3 + 2x6 = 9 + 12 = **21**	21	6
6	C2(0,6);C3(0,1,5);C3(0,2,4) C2(0,3,3);C2(1,1,4);C3(1,2,3); C1(2,2,2)=3C2+3C3+C1 = **7**	3P2 + 3P3 + P1 = 3x3 + 3x6 + 3 = 9 + 18 + 1 = **28**	28	7
7	C2(0,7);C3(0,1,6);C3(0,2,5)C3(0,3,4);C2(1,1,5); C3(1,2,4);C2(1,3,3);C2(2,2,3)=4C2+4C3= **8**	4P2 + 4P3 = 4x3 + 4x6 = 12 + 24 = **36**	36	8
8	C2(0,8);C3(0,1,7);C3(0,2,6);C3(0,3,5);C2(0,4,4);C2(1,1,6);C3(1,2,5);C3(1,3,4);C2(2,2,4);C2(2,3,3) = 5C2 + 5C3 = **10**	5P2 + 5P3 = 5x3 + 5x6 = 15 + 30 = **45**	45	9
9	C2(0,9);C3(0,1,8);C3(0,2,7) C3(0,3,6); C3(0,4,5);C2(1,1,7);C3(1,2,6);C3(1,3,5);C2(1,4,4);C2(2,2,5);C3(2,3,4);C1(3,3,3) = 4C2+7C3+C1 = **12**	4P2 + 7P3 + P1 = 4x3 + 7x6 + 1 = 12 + 42 + 1 = **55**	55	10
10	C3(0,1,9);C3(0,2,8);C3(0,3,7);C3(0,4,6);C2(0,5,5);C2(1,1,8);C3(1,2,7);C3(1,3,6);C31,4,5);C2(2,2,6);C3(2,3,5);C2(2,4,4);C2(3,3,4) =5C2+8C3 = **13**	5P2 + 8P3 = 5x3 + 8x6 = 15 + 48 = **63**	63	8
11	C3(0,2,9);C3(0,3,8);C3(0,4,7);C3(0,5,6);C2(1,1,9);C3(1,2,8);C3(1,3,7);C3(1,4,6);C2(1,5,5);C2(2,2,7);C3(2,3,6);C3(2,4,5);C2(3,3,5);C2(3,4,4) = 5C2 + 9C3 = **14**	5P2 + 9P3 = 5x3 + 9x6 = 15 + 54 = **69**	69	6
12	C3(0,3,9);C3(0,4,8);C3(0,5,7);C2(0,6,6);C3(1,2,9);C3(1,3,8);C3(1,4,7);C3(1,5,6);C2(2,2,8);C3(2,3,7);C3(2,4,6);C2(2,5,5);C2(3,3,6);C3(3,4,5);C1(4,4,4) = C1 + 4C2 + 10C3 = **15**	P1 + 4P2 + 10P3 = 1 + 4x3 10x6 = 1 + 12 +60 = **73**	73	4
13	C3(0,4,9);C3(0,5,8);C3(0,6,7);C3(1,3,9);C3(1,4,8);C3(1,5,7);C2(1,6,6);C2(2,2,9);C3(2,3,8);C3(2,4,7);C3(2,5,6);C2(3,3,7);C3(3,4,6);C2(3,5,5);C2(4,4,5) = 5C2 +10C3 = **15**	5P2 + 10P3 = 5x3 + 10x6 = 15 + 60 = **75**	75	2
14	C3(0,5,9);C3(0,6,8);C2(0,7,7);C3(1,4,9);C3(1,5,8);C3(1,6,7);C3(2,3,9);C3(2,4,8);C3(2,5,7);C2(2,6,6);C2(3,3,8);C3(3,4,7);C3(3,5,6);C2(4,4,6);C2(4,5,5) = 5C2 + 10C3 = **15**	5P2 + 10P3 = 5x3 + 10x6 = 15 + 60 = **75**	75	0

Table 4 (continued).

15	C3(0,6,9);C3(0,7,8);C3(1,5,9);C3(1,6,8);C2(1,7,7);C3(2,4,9);C3(2,5,8);C3(2,6,7);C2(3,3,9);C3(3,4,8);C3(3,5,7);C2(3,6,6);C2(4,4,7);C3(4,5,6);C1(5,5,5) = C1 + 4C2 + 10C3 = **15**	P1 + 4P2 + 10P3 = 1 + 4x3 + 10x6 = 1 + 12 + 60 = **73**	73	-2
16	C3(0,7,9);C2(0,8,8);C3(1,6,9);C3(1,7,8);C3(2,5,9);C3(2,6,8);C2(2,7,7);C3(3,4,9);C3(3,5,8);C3(3,6,7);C2(4,4,8);C3(4,5,7);C2(4,6,6);C2(5,5,6) = 5C2 + 9C3 = **14**	5P2 + 9P3 = 5x3 + 9x6 = 15 + 54 = **69**	69	-4
17	C3(0,8,9);C3(1,7,9);C2(1,8,8);C3(2,6,9);C3(2,7,8);C3(3,5,9);C3(3,6,8);C2(3,7,7);C2(4,4,9);C3(4,5,8);C3(4,6,7);C2(5,5,7);C2(5,6,6) = 5C2 + 8C3 = **13**	5P2 + 8P3 = 5x3 + 8x6 = 15 + 48 = **63**	63	-6
18	C2(0,9,9);C3(1,8,9);C3(2,7,9);C2(2,8,8);C3(3,6,9);C3(3,7,8);C3(4,5,9);C3(4,6,8);C2(4,7,7);C2(5,5,8);C3(5,6,7);C1(6,6,6) = C1+4C2+7C3 = **12**	P1 + 4P2 7P3 = 1 + 4x3 + 7x6 = 1 + 12 + 42 = **55**	55	-8
19	C2(1,9,9);C3(2,8,9);C3(3,7,9);C2(3,8,8);C3(4,6,9);C3(4,7,8);C2(5,5,9);C3(5,6,8);C2(5,7,7);C2(6,6,7) = 5C2 + 5C3 = **10**	5P2 + 5P3 = 5x3 +5x6 = 15 + 30 = **45**	45	-10
20	C2(2,9,9);C3(3,8,9);C3(4,7,9);C2(4,8,8);C3(5,6,9);C3(5,7,8);C2(6,6,8);C2(6,7,7) = 4C2 + 4C3 = **8**	4P2 + 4P3 = 4x3 + 4x6 = 12 + 24 = **36**	36	-9
21	C2(3,9,9);C3(4,8,9);C3(5,7,9);C2(5,8,8);C2(6,6,9);C3(6,7,8);C1(7,7,7) = C1+3C2+3C3 = **7**	P1 + 3P2 + 3P3 = 1 + 3x3 + 3x 6 = 1 + 9 + 18 = **28**	28	-8
22	C2(4,9,9);C3(5,8,9);C3(6,7,9);C2(6,8,8);C2(7,7,8) = 3C2 + 2C3 = **5**	3P2 + 2P3 = 3x3 + 2x6 = 9 + 12 = **21**	21	-7
23	C2(5,9,9);C3(6,8,9);C2(7,7,9);C2(7,8,8) = 3C2 + C3 = **4**	3P2 + P3 = 3x3 + 6 = 9 + 6 = **15**	15	-6
24	C2(6,9,9);C3(7,8,9);C1(8,8,8) = C1 + C2 + C3 = **3**	P1 + P2 + P3 = 1 + 3 + 6 = **10**	10	-5
25	C2(7,9,9);C2(8,8,9) = 2C2 = **2**	2P2 = 2x3 = **6**	6	-4
26	C2(8,9,9) = **1**	P2 = **3**	3	-3
27	C1(9,9,9) = **1**	P1 = 1	1	-2
	Total combinations, G = 220	**Total permutations, F =1000**	**1000**	

ferent (and 2 same) such as 4456, 2377, etc; and C4 for all 4 different digits such as 1230, 5687, etc; and let P0, P1, P2, P3 and P4 denote the corresponding possible permutations, then it is clear that P0 = 1, P1 = 4, P2 = 6, P3 = 12 and P4 = 24. The symbols G (groups) and F (frequencies) will denote total combinations and total permutations respectively as in the case of the 3-digit whole numbers already explained above.

A full analysis of the 4-digit case is shown below in Table 5.

Concluding Comments

It may now be stated that we have more or less adequately answered our First Original Question about the totals of 4-digit car registration numbers ranging for each alphabet from 0001 to 9999. (Interested readers are in-

Table 5.

Digit sums (DS)	Possible Combinations, C with total = G	Possible Permutations, P with total = F	dF	ddF
0	C0(0000) = **1**	P0 = **1**		
1	C1(0001) = **1**	P1 = **4**	3	
2	C1(0002); C2(0011) = **2**	P1+P2 = 4+6 = **10**	6	3
3	2C1(0003,0111)**;** C3(0012) = **3**	2P1 + P3 = 2x4 + 12 = **20**	10	4
4	C0(1111); C1(0004); C2(0022); 2C3(0013,0112) = **5**	P0+P1+P2+2P3 = 1 + 4 + 6 + 2x12 = **35**	15	5
5	2C1(0005,1112); 4C3(0014, 0023, 0113,0122) = **6**	2P1 + 4P3 = 2x4 + 4x12 = **56**	21	6
6	3C1(0006,0222,1113); 2C2(0033, 1122); 3C3(0015, 0024, 0114) C4(0123) = **9**	3P1 + 2P2 + 3P3 + P4 = 3x4 + 2x6 + 3x12 + 24 = **84**	28	7
7	3C1(0007,1114,1,222); 7C3(0016, 0035,0034,0115,0133, 0223, 1123) ; C4(0124) = **11**	3P1 + 7P3 + P4 = 3x4 + 7x12 + 24 = **120**	36	8
8	C0(2222); 2C1(0008,1115); 2C2 (0044,1133);8C3(0017,0026,0035, 0116,0224,0233,1124,1223); 2C4 (0125,0134) = **15**	P0 + 2P1 + 2P2 + 8P3 + 2P4 = 1+2x4 + 2x6 + 8x12 + 2x24 = 1 + 8 + 12 + 96 + 48 = **165**	45	9
9	4C1(0009,1116,2223,0333); 11C3 (0018,0027,0036,0045,0117,0144, 0225,1125,1134,1224,1233); 3C4 (0126,0135,0234) = **18**	4P1+11P3+3P4=4x4 + 11x12 + 3x24 = 16 + 132 + 72 = **220**	55	10
10	3C1(1117,1333,2224); 3C2(0055, 1144,2233);11C3(0019,0028,0037, 0046,0118,0226,0244,0334,1126, 1135,1225); 5C4(0127,0136,0145, 0235,1234) = **22**	3P1 + 3P2 + 11P3 + 5P4 = 3x4 + 3x6 + 11x12 + 5x24 = 12 + 18 + 132 + 120 = **282**	62	7
11	3C1(1118,2225,2333); 16C3(0029, 0038,0047,0056,0119,0155,0227,0335,0344, 1127,1136,1145,1226,1244, 1334,2234); 6C4(0128,0137,0146, 0236,0245,1235) = **25**	3P1 + 16P3 + 6P4 = 3x4 + 16x12 + 6x24 = 12 + 192 + 144 = **348**	66	4
12	C0(3333); 3C1(0444, 1119, 2226); 3C2(0066,1155,2244); 14C3(0039, 0048,0057,0228,0255,0336,1128, 1137,1146,1227,1335,1344,2235, 2334); 9C4(0129,0138,0147,0156, 0237, 0246, 0345,1236,1245) = **30**	P0 + 3P1 + 3P2 + 14P3 + 9P4 = 1 + 3x4 +3x6 +14x12+9x24 = 1 + 12 + 18 + 168 + 216 = **415**	67	1
13	3C1(1444,2227,3334); 19C3(0048, 0058,0067,0166,0229, 0337,0355, 0445,1129,1138,1147, 1156,1228, 1255,1336,2236,2245, 2335,2344); 10C4(0139,0148,0157,0238,0247, 0256,0346,1237,1246,1345) = **32**	3P1 + 19P3 + 10P4 = 3x4 + 19x12 + 10x24 = 12 + 228 +240 = **480**	65	=2

Table 5 (continued).

14	3C1(2228,2444,3335); 4C2 (0077, 1166,2255,3344); 16C3(0059,0068, 0266,0338,0446,0455,1139,1148, 1157,1229,1337,1355,1445,2237, 2246,2336); 13C4(0149,0158,0167, 0239,0248,0257,0347,0356,1238, 1247,1256,1346,2345) = **36**	3P1 + 4P2 + 16P3 + 13P4 = 3x4 + 4x6 + 16x12 + 13x24 = 12 + 24 + 192 +312 = **540**	60	-5
15	4C1(0555,2229,3336,3444); 20C3 (0069,0078,0177,0339,0366,0447, 1149,1158,1167,1266,1338,1446, 1455,2238,2247,2256,2337,2355, 2445,3345); 14C4(0159,0168,0249, 0258,0267,0348,0357,0456,1239, 1248,1257,1347,1356,2346) = **38**	4P1 + 20P3 + 14P4 = 4x4 + 20x12 + 14x24 = 16 + 240 + 336 = **592**	52	-8
16	C0(4444); 2C1(1555,3337); 4C2 (0088,1177,2266,3355);18C3(0079, 0277,0448,0466,0556,1159.1168, 1339,1366,1447,2239,2248,2257, 2338, 2446, 2455,3346,3445); 16C4 (0169,0178,0259,0268,0349,0358, 0367,0457.1249,1258,1267,1348, 1357,1456,2347,2356) = **41**	P0 +2P1 + 4P2 +18P3 + 16P4 = 1+2x4+4x6+18x12+16x24 = 1 + 8 + 24 + 216 + 384 = **633**	41	-11
17	3C1(2555,3338,4445); 22C3 (0089, 0188,0377,0449,0557,0566,1169, 1178,1277,1448,1466,1556,2249, 2258,2267,2339,2366,2447,3347, 3356,3446,3455); 16C4(0179,0269, 0278,0359,0368.0458,0467,1259, 1268,1349,1358,1367,1457,2348, 2357,2456) = **41**	3P1 + 22P3 + 16P4 = 3x4 + 22x12 + 16x24 = 12 + 264 + 384 = **660**	27	-14
18	4C1(0666,3339,3555,4446); 5C2 (0099,1188,2277,3366,4455); 16C3 (0288,0477,0558,1179,1377,1449, 1557,1566,2259,2268,2448,2466, 2556, 3348,3357,3447); 18C4(0189, 0279,0369,0378,0459,0468,0567, 1269,1278,1359,1368,1458,1467, 2349,2358,2367,2457,3456) = **43**	4P1 + 5P2 + 16P3 + 18P4 = 4x4 + 5x6 + 16x12 + 18P4 = 16 + 30 + 192 + 432 = **670**	10	-17
19	**41**	**660**	-10	-20
20	**41**	**633**	-27	-17
21	**38**	**592**	-41	-14
22	**36**	**540**	-52	=11
23	**32**	**480**	-60	-8
24	**30**	**415**	-65	-5
25	**25**	**348**	-67	-2
26	**22**	**282**	-66	+1

Table 5 (continued).

27	**18**	**220**	-62	+4
28	**15**	**165**	-55	+7
29	**11**	**120**	-45	+10
30	**9**	**84**	-36	+9
31	**6**	**56**	-28	+8
32	**5**	**35**	-21	+7
33	**3**	**20**	-15	+6
34	**2**	**10**	-10	+5
35	**1**	**4**	-6	+4
36	**1**	**1**	-3	+3
	Total combinations, G = 715	**Total permutations, F=10,000**		

vited to fill in the blanks in the above Table 5). In our systematic mathematical analysis, we have also solved the cases of 1-digit, 2-digit and 3-digit numbers in relation to their digit sums, their statistical distributions along with various other related characteristics.

It is found that the distributions of both G and F (these symbols have already been defined) are symmetrical about DS = 18, half of 36 (maximum possible DS) in the case of 4-digit numbers; and that the total of G and F are 715 and 10,000 respectively.

It is also clear that there are several avenues open for further research and analysis in respect of the results of our studies so far, such as an investigation into the pattern, characteristics and behaviour of dF, ddF, dddF, etc for the 3-digit and the 4-digit cases along with their comparisons; and what conclusions can be drawn from such studies. But we will leave these matters for the next stage of our research. In fact, I have already done some interesting analysis in this direction.

One may well ask: Is it possible, from what we have observed so far, to deduce a general formula for the analysis of digit sums of n-digit whole numbers when n takes values such as 5, 6, 7, etc. that will enable one to avoid the heavy computational work involved? Or inversely, is it possible to deduce the patterns and distributions of digit sums of higher digit whole numbers through analysis of the results already calculated without going into the burden of heavy computations? The next task is thus to seek answers to these questions and then, if possible, to extend it to the general case.

The last question I would like to raise is this: Is there any similar work of this nature in mathematical literature known to any of the readers? If so, I would be most interested to be informed. As far as I know, this paper is the only research and analysis of this nature in existence. It would be most appreciated if anyone may throw some light on this last question.

Corrigendum

The paper "Recent status of threatened birds of Mizoram" by Lalthanzara in *Sci Vis* 10 (4), 168-169 should be categorized as 'Report', not 'Original Research', but *only in the print version*. We regret on the copy-paste error.